International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 4 Apr 2022, pp: 559-572 www.ijaem.net ISSN: 2395-5252

Design and Review of Snake Robot and Locomotion

Mr. Varad Nandkishor Dhande, Mr. Shreyash Abhay Kamble, Mr. Shantanu Vishal Kulkarni, Mr. Sachin Kumar Gautam, Mr. Ganesh Baban Kolekar

Department of Mechatronics Engineering New Horizon Institute of Technology & Management, Thane University ofMumbai, India
Department of Mechatronic Engineering New Horizon Institute of Technology & Management, Thane University ofMumbai, India
Department of Mechatronic Engineering New Horizon Institute of Technology & Management, Thane University ofMumbai, India
Department of Mechatronic Engineering New Horizon Institute of Technology & Management, Thane University of Mumbai, India
Professor of Department of Mechatronic Engineering New Horizon Institute of Technology & Management, Thane University ofMumbai, India

Submitted: 01-04-2022 Revised: 12-04-2022 Accepted: 15-04-2022

ABSTRACT— Robots are expected to be new tools for the operations and observations in the extreme environments where humans have difficulties in direct access. One of the important matters to realize mobile robots for extreme environments is to establish systems in their structures which are strong enough to disturbances. This work aims at the study of a snake robot for the surveillance and spying purpose operations in remote area as well as for the military purpose. A biologically inspired robot with various motion patterns is taken into consideration. An important problem seen here in the control of locomotion of robots with multiple degrees of freedom is in adapting the locomotors patterns of a snake. A wireless real time vision processing is also employed within the robot to improve its performance. Real time processing of video enables proper and efficient control towards obstacle avoidance pattern of the robot. This ensures that the locomotion of the robot is in a bio-inspired highly efficient path towards the target.

Keywords: Collision-free behavior, neural oscillator, snake locomotion, steering, real time vision processing

I. INTRODUCTION

The wheel is an amazing invention, but it does not rolleverywhere.Wheeledmechanismsconstitutethebackboneof most ground-based means of transportation. On relativelysmooth surfaces, such mechanisms can achieve high speedsand have good steering ability. Unfortunately, rougher terrainmakes it harder, if not impossible, for such mechanisms tomove. In nature, the snake is one of the creatures that exhibitexcellentmobilityinvariousterrains.Itisableto movethroughnarrowpassagesandclimbonroughground.Thispropertyofmobilityisattemptedrecreatedinrobotsthatlookandmovelikesnakes.Snakerobotsmostoftenhaveahighnumberof degrees of freedom (DOF) and they are able to locomotewithoutusingactivewheelsorlegs.
Snakerobotssuitawiderangeofapplications.Oneofmanyexamplesisrescuemissionsinearthquakeareas.The snakerobotcouldcrawlthroughdestroyedbuildingslookingforpeople.Itcouldalsocarrysmallamountsoffoodorwatertopeopletrappedbythebuildingpriortothearrivalofrescuepersonnel.Thesnakerobotcanalsobeused forsurveillanceand maintenance of complex and possibly dangerous structuressuchasnuclearplantsorpipelines.Inacity,itcouldinspectthe sewer system looking for leaks or aiding fire-fighters. Also,snakerobotswithoneendfixedtoabasemaybeusedasarobotmanipulatorwhichcanreachhard-to-get-toplaces.
Compared to wheeled and legged mobile mechanisms, thesnake robot offers high stability and good terrainability.

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 4 Apr 2022, pp: 559-572 www.ijaem.net ISSN: 2395-5252

Theexteriorcanbecompletelysealedtokeepdustandfluidsout.Duetohighredundanceandmodularity,thesnakerobot

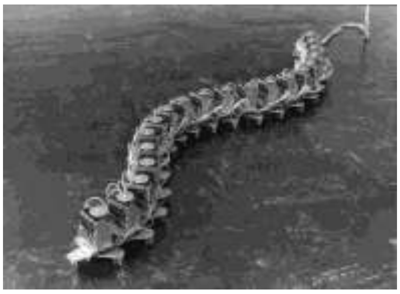

Fig. 1.The Active Cord Mechanism model ACM III [2].

Is robust to mechanical failure. The downside is its limitedpayloadcapacity,poorpowerefficiencyandaverylargenumberofdegreesoffreedomthathavetobecontrolled.
Thefirstqualitativeresearchonsnakelocomotionwasdoneby J. Gray in 1946 [1]. The first working biologically inspiredserpentinerobotwasmadebyShigeoHirosein 1972[2].He presented a two-meter-long serpentine robot with twentyrevolute1DOFjointscalledtheActiveCordMechanismmodel ACM III shown in Fig. 1. Passive casters were put onthe underside of the robot. Forward motion was obtained bymovingthejointstotheleftandrightinselectedpatterns.
SinceHirosepresentedhis"ActiveCordMechanism", manymulti-link articulated robots intended for crawling locomotionhavebeendevelopedandtheyhavebeencalledbymanynames.Someexamplesare:multi-linkmobilerobot[3],snake-like or snake robot [4]–[10], hyper-redundant robot [11] andG-snake [12]. To emphasize that this paper deals with robotsthat mainly resemble locomotion of snakes, the term "snakerobot"willbeemployed.Thesnakerobotspresentedareimplementedeitherwithpassivewheels[3],[13]–[15]orwithout wheels [16]–[21]. The joints are mostly revolute, butextensible(prismatic)jointsarealsoemployed[17],[22].
Motion patterns of snakes, inchworms and caterpillars areused as an inspiration for how the snake robots should move.Mathematicalmodelsofthesnakerobotsareneededtoanalyzethe motion patterns and to simulate their motion. Because ofthehighnumberofDOF,theconstructionofsuchmodelsisa challenge. During the last ten to fifteen years, the publishedliterature on snake robots has increased vastly, and the purposeofthisarticleistoprovideanoverviewandcomparisonofthevariousmathematicalmodelsandlocomotionprinciples of snake robots presented during this period. The relationshipbetween snake robot design and the choice of gait is outlined,and some recent results on locomotion patterns are given. Wealso provide an introduction to the source of inspiration ofsnake robots: biologically inspired crawling locomotion.
Somepossiblyadvantageousbiologicalmotionpatternswhicharenot yet implemented are mentioned. Selected mathematicalmodels will be more thoroughly presented. The choices ofsensorsandactuatorswillnotbediscussed.
This paper is arranged as follows: Sec. II gives a shortintroduction to snakes and biological, crawling locomotion.Various mathematicalmodels ofsnake robots arepresentedin Sec. III. Sec. IV gives examples of control signals used toobtainlocomotionwhileconclusionsandsuggestionstofurtherresearcharegiveninSec.V.

II. BIOLOGICALSNAKESANDINCHWORMS

Biologicalsnakes,inchwormsandcaterpillarsarethesourceof inspiration for most of the robots dealt with in this paper.Wewillthereforestartwithashortintroductiontosnakephysiology and snake locomotion. Unless otherwise specified,thecontentsinthissectionarebasedon[23]–[25].

A. SnakeSkeleton

The skeleton of a snake often consists of at least 130 ver-tebrae,andcanexceed400vertebrae.Therangeofmovement betweeneachjointislimitedtobetween10°and20°for rotationfromsidetoside,andtoafewdegreesofrotation when moving up and down. A large total curvature of thesnake body is still possible because of the high number ofvertebrae.
A very small rotation is also possible around the directionalong the snake body. This property is employed when thesnakelocomotesbysidewinding.

B. SnakeSkin

Sincesnakeshavenolegs,theskinsurfaceplaysanimportant role in snake locomotion [24]. The snake shouldexperiencelittlefrictionwhenslidingforwards, butgreatfriction when pushed backwards. The skin is usually coveredwithscaleswithtinyindentationswhichfacilit

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 4 Apr 2022, pp: 559-572 www.ijaem.net ISSN: 2395-5252

ateforwardlocomotion.Thescalesformanedgetothebellyduringmotionwhich results in that the friction between the underside of thesnake and the ground is higher transversal to the snake bodythanalongit[13].

C. Locomotion-TheSourceofInspirationforSnakeRobots

Mostmotionpatternsusedbysnakerobotstolocomoteare inspired by locomotion of snakes, but also inchworms andcaterpillars.Therelevantmotionpatternsofsuchcreatureswillbeoutlinedinthefollowing.

1) Lateral Undulation:Lateral undulation (also denotedserpentine crawling) is a continuous movement of the entirebodyofthesnakerelativetotheground.Locomotionisobtained by propagating waves from the front to the rear of thesnake while exploiting roughness in the terrain. Every part ofthe body passes the same part of the ground ideally leaving asinglesinus-liketrackasillustratedinFig.2(a).Thebodyofthesnakeneedstotouchthegroundatthreepointstoobtain a continuous forward motion. Two points are needed togenerateforces.Thethirdpointisusedtobalancetheforces

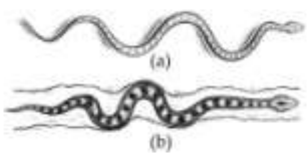

Fig.2.(a)Lateralundulationand(b)concertinalocomotion[23].BypermissionofCassellIllustrated.

such that they act forwards. To prevent lateral slipping whilelocomoting, the snake "digs in" to the ground with help of theedge described in Sec. II-B. It also uses contours such as rocksonthegroundtopushagainst.

The efficiency of lateral undulation is mainly based on twofactors.1)Thecontouroftheground.Themorecontouredthe ground, the more efficient the locomotion. 2) The ratiobetween the length of the snake and its circumference. Thefastest snakes have a length that is no longer than 10 to 13times their circumference. Speeds up to 11 km/h have beenobservedinroughterrains.

2) Concertina Locomotion: A concertina is a small accor-dion instrument. The name is used in snake locomotion toindicate that the snake stretches and curves its body to moveforward. The folded part is kept at a fixed position while therest of the body is either pushed or pulled forward as shown inFig. 2 (b). Then, the two parts switch roles. Forward motion isobtained when the force needed to push back the fixed part ofthe snake body is higher than the friction forces on the movingpartofthebody.

3)

Concertina locomotion is employed when the snake movesthroughnarrowpassagessuchaspipesoralongbranches.Ifthe path is too narrow compared to the diameter and curvingcapacityofthesnake,thesnakeisunabletolocomote.

4) Sidewinding Locomotion:Sidewinding is probably themost astonishing gait to observe and is mostly used by snakesin the desert. The snake lifts and curves its body leaving short,parallelmarksonthegroundwhilemovingatanin clinedangleas shown in Fig. 3. Unlike lateral undulation, there is a briefstaticcontactbetweenthebodyofthesnakeandthe ground.

5)

Sidewindingisusuallyemployedonsurfaceswithlowshearsuchassand.Thesnakescanreachvelocitiesupto3 km/h.

6) OtherSnakeGaits:Snakesalsohavegaitsthat areemployedinspecialsituationsorbycertainspecies. Theseare e.g. rectilinear crawling, burrowing, jumping, sinus-lifting,skidding, swimming, and climbing. The latter four, which areormaybeusedforsnakerobotsareasfollows.

Sinus-lifting is a modification of lateral undulation wherepartsofthetrunkareliftedtoavoidlateralslippage andto

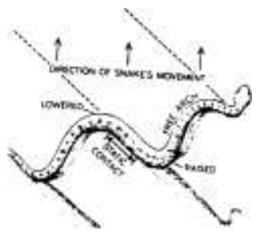

Fig.3.Sidewindinglocomotion[26].

optimize propulsive force [13]. The gait is employed for highspeeds.

A variation of lateral undulation is called skidding (alsodenotedslidepushing)andisemployedwhenmovingpastlow-friction surfaces. The snake rests its head on the ground andthen sends aflexion wavedownthrough itsbody.Thisisrepeated in a zigzag pattern and is a very inefficient way

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 4 Apr 2022, pp: 559-572 www.ijaem.net ISSN: 2395-5252

tolocomote.
Almost all snakes can swim. They move forward by undu-latinglaterallylikeaneel.
Long and thin bodied snakes can climb trees by verticallateral undulation. Parts of their body hang freely in the air,whilebranchesareusedassupport.
7) Inchworm and Caterpillar Locomotion:Inchworms lo-comote by curving their body grabbing the ground with itsfrontlegswhiletherearendispulledforward.Therear legsthengrabthegroundandtheinchwormstraightens.
Caterpillars send a vertical travelling wave through their bodyfrom the end to the front. Small legs give friction while on theground.

III. DESIGNANDMATHEMATICALMODELING

Themathematicalmodelofasnakerobot,ofcourse,depends on its design. To categorize the different snake robotdesignswerecognizecertainbasicproperties:1)Typeofjoints, 2)numberofdegreesoffreedom(DOF),and3)withorwithout wheels. Most snake robots consist of links connectedbyrevolutejointswithoneortwoDOF.Onsomerobots,the links are extensible (i.e. prismatic joints). To achieve thedesired frictional property for lateral undulation mentioned inSec. II, some snake robots are equipped with passive wheels.When wheels are employed, the dynamics of the interactionbetweentherobotandthegroundsurfaceisoftenignored.Ifnowheels are attached, this friction force needs to be consideredforsome,butnotall,gaits(seeSec.IV).
Inthefollowing,themathematicalmodelingofthedifferentsnakerobotsisdividedintokinematicsanddynamics.

A. Kinematics

The kinematics describes the geometrical aspect of motion.Different modeling techniques ranging from classical meth-ods such as the Denavit-Hartenberg convention (see [27]) tospecialized methods for hyper-redundant structures (structureswithahighnumberofDOF)havebeenemployed.Thefollowing subsections will elaborate on the different modelingtechniques.

1) Denavit-Hartenberg:TheDenavit-Hartenberg(D-H)conventionisawell-establishedmethodfordescribingthepositionandorientationofeachjointofarobotmanipulatorwith respect to a (usually fixed) base frame. Different solutionsarepresentedtodealwiththefactthebaseisnotfixedonasnakerobotin[28],[29].
Reference [29] presents a snake robot that is made of 9equal modules. Each module consists of seven revolute 1 DOFjoints which are connected by links of equal length. Threejoints and four joints have the axis of rotation perpendicular tothe horizontal and vertical plane, respectively. Each module isparameterized with the D-H convention. A modification to theconvention has been proposed by placing the base coordinatesystem on the first motionless link of the part of structurewhichisinmotion.Hence,thelinksinmotionaredescribed inaninertialframe.Thesnakerobotin[29]movesonlyfourorfivemodules simultaneously,so givingthepositionand orientation relative to the first motionless link preventstraversing through the complete structure to obtain positionsandorientationsinaninertialframe.
The locomotion scheme in [28] is based on constant jointmovement, so we have to traverse through the whole structureand hence the approach in [29] will not simplify the mathe-matical structure. Therefore, a virtual structure for orientationand position (VSOP) is introduced to be able to describe thekinematics of the snake robot in an inertial reference frame.Reference [28] presents a snake robot with 5 revolute 2 DOFjoints.TheVSOPdescribesthetrailinglinkofthesnakerobot in an inertial reference frame by 3 orthogonal prismaticjoints and 3 orthogonal revolute joints which represent theposition and orientation, respectively. The joints are connectedbylinkswithnomass.ByemployingtheVSOPintheDenavit-Hartenberg convention, the position and orientation of eachjointisgiveninaninertialcoordinatesystem.

2) A Backbone Curve (and its Reference Set):Instead ofstarting by finding the position and orientation of each jointdirectlyaswiththeDenavit-Hartenbergconvention [30],abackbonecurvemaybeemployed.Thebackbone curveisdefined in [11] as "a piecewise continuous curve that capturestheimportantmacroscopicgeometricfeatures ofahyper-redundant robot" and it typically runs through the spine of thesnake robot. A set of orthonormal reference frames are foundalong the backbone curve to specify the actual snake robotconfiguration. The backbone curve parametrization togetherwithanassociatedsetoforthonormalreference framesiscalleda backbone curve reference set and allows for snake robotsbuiltfrombothprismaticandrevolutejoints[31].
The problem of determining joint angles of a robot ma-nipulator given the end-effector position is called the inversekinematics problem. For hyper-

redundant manipulators (suchas snakerobots)this is averycomputationallydemandingtask. When the backbone curve is employed, the problem isreduced to determining the proper time varying behavior ofthe backbone reference set. The method of backbone curveshas notbeen found togetherwithmodeling ofdynamics, butis rather a method for abstraction and understanding of thegeometric aspects of snake robot motion planning where thedynamicsmaybeneglected.

3) Nonholonomic Constraints and Snake Robots with Pas-sive Caster Wheels:The key to snake robot locomotion is tocontinuouslychangetheshapeoftherobot.Thisisachievedbyrotation and/or elongation of its joints. References [12], [14]both present kinematic approaches on how to link the changesininternalconfigurationtothenetpositionchangeoftherobot.The relation is found by utilizing nonholonomic constraintsand differential geometry such as connections. Reference [14]employs Hirose's Active Cord Mechanism Model 3 (ACM III)as an example which will be explained in the following.

ThefirstthreepairofwheelsofACMIIIareillustratedin Fig.

4. The five joint angles φ_1, φ_2, φ_3, ψ_1,and $\psi_3$are controlledinputs.Thekinematicnonholonomicconstraintsarerealizedbyaddingpassivecasterwheelsonthesnakerobotandmaybewritteninthe form

$$\dot{x}_i\sin(\varphi_i)-\dot{y}_i\cos(\varphi_i)=0 \quad (1)$$

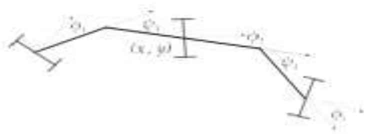

Fig.4.ThefirstthreelinksoftheACMIII[14].

where($\dot{x}_i,\dot{y}_i$)isthevelocityofthecenterofmassand$\varphi_i$is theangleofthejointwhichthewheelsareattachedto.Moreonnonholonomicsystemsarefoundin[32],[33].Thewheels

areassumednottoslipandthereforerealizeanidealversionof the frictional properties of the snake skin as mentioned in Sec.II-B.

A*localform*A of a *connection* providestherelationbetweentheshapechangesofthesnakerobotanditsnetlocomotion:

$$g^{-1}\dot{g}=-A(r)\dot{r} \quad (2)$$

where$r$istheshapevariablesand$g$ $SE(2)$givesthe overall position and orientation of the snake robot [14]. Theconnection provides understanding of how shape changes cangeneratelocomotionandcanevenbeusedforcontrolla-bilitytests[34].Thesimpleformof(2)isdependentonthe kinematic constraints breaking all the symmetries of theLagrangianfunctionwhichmayraisedynamicconstraints.This is achieved, with the ACM III as an example by using thefirstthreesegmentstodefinethenetmotionofthesnakerobot.These segments define the path which is to be followed by theremaining segments due to the nonholonomic constraints onthewheels.

Asopposedtothebackbonecurvereferenceset,thismodeling techniquemay also includethedynamics aswillbedescribedinSec.III-B.2.e.

Dynamics

Thedynamicsofthesnakerobotspresentedhasbeenderived by utilizing various modeling techniques such as theNewton-Euler formulation, Lagrange functions and geometricmechanics.

For snake robots without wheels, the friction between thesnake robot and the ground affects the motion of the snakerobot significantly. Thus, for these snake robots, the dynamicsshould be modelled for locomotion patterns such as lateralundulation. For snake robots with wheels, however, the wheelsgreatlyreducethefrictionand,hence,makeitpossibletouse apurelykinematicmodeloftherobot.Themajorityoftheresultspresentedonmodelingofthedynamicshavet acceleration-based control algorithms. It is assumed that thewheelsdonotslipsideways.

A snake robot (called the SR#2) has been presented andcompared to the ACM-III in [3]. The Active Cord Mechanism(ACM) modeling assumes that the wheels do not slip. Thisnon-slippage introduces nonholonomic constraints. The SR#2model is based on holonomic framework and is hence withoutthe no-slip condition. The argument used against assuming noslip is that it is difficult to control the torques in the joints suchthattheassumptionissatisfied.SimulationsshowtheACM-IIIbuild up an error in position while following a circular path.This is not the case for SR#2, something which makes it amoreaccuratemodelforthisscenario.

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 4 Apr 2022, pp: 559-572 www.ijaem.net ISSN: 2395-5252

1) Snake Robots without Wheels:The use of wheels de-creasesterrainability[19],thuswheel-lessrobotshaveanadvantage.Asdiscussedearlier,frictionplaysasignificantrolefor wheel-less snake robots, hence it is necessary to model thedynamics and not only the kinematics for relatively high speedmotion.Anoverviewofthefrictionmodelsemployedwillfirstbegiven,thenaselectionofdynamicmodelsderivedforsnakerobotswithoutwheelswillbepresented.

a) Friction Models:The friction models presented inliteraturearebasedonaCoulomborviscous-likefrictionmodelandcanbefound,forinstance,in[36]. Aspring-ampermodelimplementsthegroundcontactforceforthe3Ddynamicmodelgivenin[28].Thecontactforceiswritten

½
0 ,z_iº0

consideredsnakerobotswithoutwheels.Inthefollowingwe

where$z_i$istheheightoflinki,$\dot{z}_i=\frac{dz}{dt}$,k istheconstant willfirstgiveashortintroductiontothenotationutilized springcoefficientoftheground,andd isaconstantdamping below, then we give a brief overview of a selection of theresultsreportedonthemodelingofdynamicsofwheeledsnakerobots,andfinallywepresenttheresultsonsnakerobotswithoutwheels.

Toeasethepresentationofthemathematical models,a coefficient that serves to damp out the oscillations

·µ
(2) i,t
c(2)¶

where$\mathbf{H}_i$=c$_{i,n}$—
unitmatrix.
i,t
(2) c
i,n
$\mathbf{e}_{i,t}\mathbf{e}^T$
—$\mathbf{I}_{2\times 2}$
,and$\mathbf{I}_{2\times 2}$isa
1DOFrevolutejointswiththeviscousfrictionmodel(5) ispresentedin[20].

Non-dimensional variables are introduced to simulate the

induced bythe spring (d, k R). Using f_{Ni} , the friction force on link i,basedonasimple,viscous-likemodel,iswrittenas

$\mathbf{f}_i=-c^{(1)}|\mathbf{f}_N|\mathbf{v}_{i,t}-c^{(1)}|\mathbf{f}_N|\mathbf{v}_{i,n}$ (4)

commonnotationforsomeofthematerialispresentedwhich
,t ii,n i
isbasedon[19],[35].Denotethemassm,length2land

Thesumofforcesactingonlinkiis$\mathbf{f}^{3D}=\sum \mathbf{f}^T f_{N_i}$.The

momentofinertia$J_i$foreachlinki=1,2,...,n.Denotetheangle$\theta_i$betweenlinkiandtheinertial(base)x-axis.Denote springcoefficientkneedstobesetveryhightoimitateasolidsurface.Hence,thetotalsystemisstiffandrequiresavery

small simulation step size to be simulated. A friction modelincluding both static and dynamic friction properties $f_{N_i} = -k z_i - d \cdot \dot{z}_i$ for, $z_i \prec 0$ (3) a 3Ddynamicmodelisfoundin[37].

The 2D anisotropic viscous friction model used in [20] canbe derived from (4) by setting f_{Ni} 1. In this case, we findthat

$\mathbf{f}_i=\mathbf{H}_i\mathbf{v}_i$, (5)

where$h_s()$, $h_i()$ R^narefunctionsfoundin[19],$\boldsymbol{\theta}=[\theta_1 \theta_n]$, **u** are the joint torques $\in$ andis an invertible matrix.Controlofthesnakerobotisnowperformedintwosteps.First,the joint torques **u** control the shape of the robot and secondthe relative angles **φ** control the average angular momentumψandposition**w**.

c)Quasi-StationaryEquationsofMotion:A2Dmodel basedontheNewton-Eulerformulationofasnakerobotwith

Theeffectofrotationalmotionofthelinksisintroducedin the two 2D friction models, one with viscous and one withCoulomb friction, presented in [19]. Both models are derivedbyintegratingtheinfinitesimalfrictionforcesonalink.

The translational part of the viscous friction model is givenby (4) with $f_{Ni} = m_i$(i.e. not a force). The total viscousfrictiontorqueduetorotationalvelocityaroundthecenterofmassoflink**i**isfoundtobe

$\tau_i = -c^{(3)}_{i,n} J_i \dot{\theta}_i$, (6)

dynamicsofthesnakerobot.Theresultingsystemofsec-ondordernonlinearequationswhichconstitutethenon-dimensionalmodelofthesnakerobotmaybecomeunsta-ble during simulation. To aid the numerical treatment, over-critical damping is introduced by setting accelerations to zero.The result is a set of quasi-stationary first-order differentialequations of motion. By employing the first-order equation fortranslational motion together with the friction model in shortform(5)thevelocityoftheheadofthesnakerobotis found to be

$$h(\boldsymbol{\varphi}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \dot{\mathbf{w}}) = \mathbf{B}\mathbf{u}, \quad (7)$$

EmployingCoulombfrictionasthefrictionmodelresultsina morecomplicated,butalsomoreaccuratemodel.Themodel

$$\mathbf{v}_H = -\left(\sum^{n}\right)^{-1} \sum^{n} H \mathbf{v}^{(rel)}_i \quad (9)$$

(4) does not include dry friction and thus the high frictionforces which may arise at low velocities are not modeled. Theresultsfromtheanalysisoftheparametersgoverningtheshapeof the snake robot during locomotion by lateral undulationweregenerallythesamefortheviscousandtheCoulombfrictionmodelin[19].

For most of the gaits simulated with the above frictionmodels, the property $c_{i,t} < c_{i,n}$has been implemented torealize the anisotropic friction property of a snake movingusinglateralundulation.Itmaybedifficulttodesignasnakerobotwith$c_{i,t} < c_{i,n}$onageneralsurface.Sidewindinghasbeenimplementedwithanisotropicfrictionmodel($c_{i,n} = c_{i,t}$)in [38] and as a purely kinematic case in [26]. Special gaitsbasedonanisotropicfrictionmodelaredetailedin[39],[40].

b) Dynamic Model with Decoupling: A five link snakerobotwith1DOFjointswasmodeledandcontrolledin[19].The robot was built and experiments performed to validatethe theoretical results. Metal skates were put on the belly toimplement$c_{ti} < c_{ni}$.

Thedynamicmodelofthesnakerobothasbeendeveloped fromtheNewton-Eulerequationsresultingintwosetsofequations:oneforttranslationalmotionofthecenterofmass**w**ofthesnakerobotandanotherfortherotationalmotionoftheangleofeachlinkgiveninaninertialframe.Thefinal equations of motion can be decoupled into two parts:shape motion and inertial locomotion. The shape motion mapsthe joint torques to joint angles while the inertial locomotionrelates the joint angles to the inertial position and orientation.This simplifiestheanalysisandsynthesis oflocomotionofthesnakerobot.Toachievedecoupling,avectorofrelative angles $\boldsymbol{\varphi}$,where$\varphi_i = \theta_i \theta_{i+1}$,anda quantity$\psi$ whichcanbethoughtofas"anaverageangularmomentum"areintroduced.Theexpressionsforshapemotionandinertialmotion,respectively,arefoundtobe

where $\mathbf{v}^{(rel)}_{head\,i}$is the velocity of link i with respect to the head.Reference [19] gives the relationship between shape changesfromjointangledeflectionandthepositionoftheCGofthesnakerobot(8).Toinvestigatelocomotionanalytically,

(9) offers an alternative approach where the direct connectionbetween velocities of each link relative to head of snakerobotandtheheadvelocityisgiven.

d) Newton-Euler Algorithm: A physical and mathemat-ical model of a snake robot with five 2 DOF joints is presentedin[28].Inadditiontotheactualsnakerobot,Avirtualstructureoforientationandposition(VSOP,see Sec.III-A.1) is included in the dynamic model. The VSOP togetherwiththesnakerobothavegeneralizedcoordinates**q**andgeneralized forces **u**. The Newton-Euler formulation and theVSOP perspective is employed,and the dynamic model iswrittenas

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{u} + \boldsymbol{\tau}_{ext} \quad (10)$$

where M is the inertia matrix, C is the Coriolis and centripetalmatrix, g ( ) is the vector of gravitational forces and torques,and $\boldsymbol{\tau}_{ext}$is the vector including the external forces. The matri-ces are detailed in [38]. While (10) has not been employed fordynamic analysis, analytical expressions for the

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 4 Apr 2022, pp: 559-572 www.ijaem.net ISSN: 2395-5252

joint torquesand head configuration of a 3D snake robot model deductedfromtheNewton-Eulermethodsareshownin[37]. TheLagrangeandtheNewton-Eulermethodaresimilarin that the expression obtained by the Lagrange method isfound by running through the Newton-Euler algorithm once.Since the Newton-Euler algorithm deals with the mathematicalmodelasarecursivealgorithm,itisamore efficientframeworkforsimulationthantheLagrangemethodforlargemodels[36].

e) Lagrange's Equations:Research on robots that re-semble snakes are not only limited to land-based locomotion.Papersregardinganguilliform(eel-like)locomotionhavealso

f) beenpublished[21],[41]–[43].Afivelink2Dsnakerobot

$$h_i^{3} \; \dot{\psi},\ddot{\psi},\boldsymbol{\theta},\dot{\mathbf{w}},\ddot{\mathbf{w}},\dot{\boldsymbol{\varphi}} = 0 \qquad (8)$$

(calledtheREELII)with1DOFrevolutejoints,whichwillbeusedasanexampleinthefollowing,hasbeenmodeledand

experimented on in [21]. Motion planning for such a robotconsists of first building up the momentum to the snake robotand thensteeringthe robot to itsdesiredlocation.Hence,itis convenient that the mathematical model includes an explicitexpression for the momentum. The model is formulated forLagrange'sequationsandissummarizedinthefollowing.

The fact that the energy of the system and the frictionalforces acting on the system are invariant with respect to theposition and orientation of the snake robot (the system exhibitsLiegroupssymmetries),isexploitedtosimplifythemathematical model. The assumption that the joint angles are controlleddirectly (The same as saying that the dynamics (7) is ignored)yield two sets of resulting equations. The first equation relatesthe velocity ofthe snake robotto its internal shape changesand is similar to (2) given in Sec. III-A.3 except for the lockedinertia tensor I (r) and generalized momentum vector p thathave been added (we have a case of mixed constraints withboth kinematic and dynamic constraints). The dynamics of thesystem is described by the generalized momentum equationwhich is the second set of resulting equations. The generalizedmomentumpisassociatedwiththemomentumalongthe directions allowed by the kinematic constraints. A thorough explanation of the equations is found in [44].

B. Architecture

The overall system architecture is described here. Each of the following sections gives a brief idea on each module and its functions. The input to the system comprises of a streaming a video from a camera installed on head of the snake robot and sensory signals placed onto a cellphone. The video gives an account of the environment in which the snake robot is moving. It captures the scene at particular instants of time and then undergoes processing to detect height of the obstacles in the scene. The IR sensor provide the distance of the robot's body to the obstacles. This information gives an idea of whether the obstacle in the video is nearby or far away.

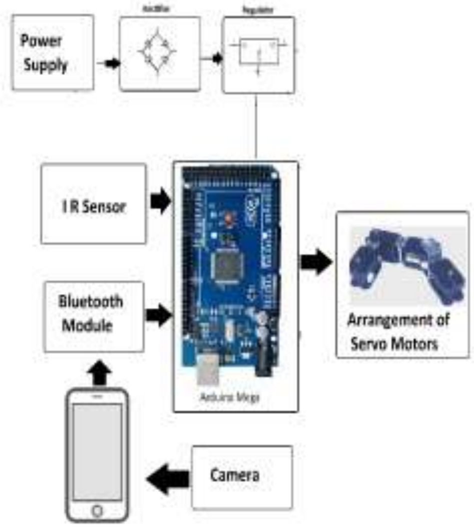

Fig. 5. Architecture of Snake Robot [40].

The input parameters are then fed to an Arduino Uno micro-controller which provides the controller action to the snake robot. The mechanical structure of the snake robot is controlled by using Bluetooth module which provides controller action via Arduino Uno micro-controller and also provides the video processing onto the cellphone module.

IV. SNAKE ROBOT LOCOMOTION

A variety of approaches on how to make a snake robotlocomote have been proposed. In most of the motion patternsor 'gaits' used for locomotion, we find a distinct resemblanceto the undulating locomotion of biological snakes or worms,but the motion patterns may be changed to compensate for thefact that the snake robots do not have the exact same anatomyas biological snakes, inchworms or caterpillars. Early studiesof snake

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 4 Apr 2022, pp: 559-572 www.ijaem.net ISSN: 2395-5252

locomotion were given in [1]. Later, a mathematicaldescription of the serpentine motion of snakes was presented[13].Anoverviewofgaitsthathavebeenimplementedisfoundin Table I where we see that lateral undulation is the mostcommon.

Dependingonhowthesnakerobotismodeled and/orconstructed, the mathematical expression for the gait varies. Adescription of the joint reference signals for lateral undulationand a short note on sidewinding will first be given, then otherlocomotionschemeswillbediscussed.

A. SnakeRobotLocomotionbyLateralUndulationandSidewinding

Lateralundulationisimplementedasasine-likewavepropagatingdownthebodyofthesnakerobotfromtheheadtothetail.Reference[13]presentsaformulaforsuchacurve called the serpenoid curve where the curvature changessmoothly.Thesnakerobotlocomotesbyfollowingthetraceofthecurve.Theserpenoidcurveisshown inFigure5and

TABLEI. OVERVIEW OF GAITS

Gait	Withpassivewheels	Withoutwheels
Concertina		[16][a]
Lateralundulation	[3],[14],[15],[17],[18],[35],[45]–[47]	[6],[19]–[21],[28],[41],[48]
Sidewinding		[26],[28]
Inchworm/Caterpillar		[17],[22],[29],[49]
Climbing		[50],[51]

[a]Usesfrictionfromsolenoidsthatareliftedandlowered

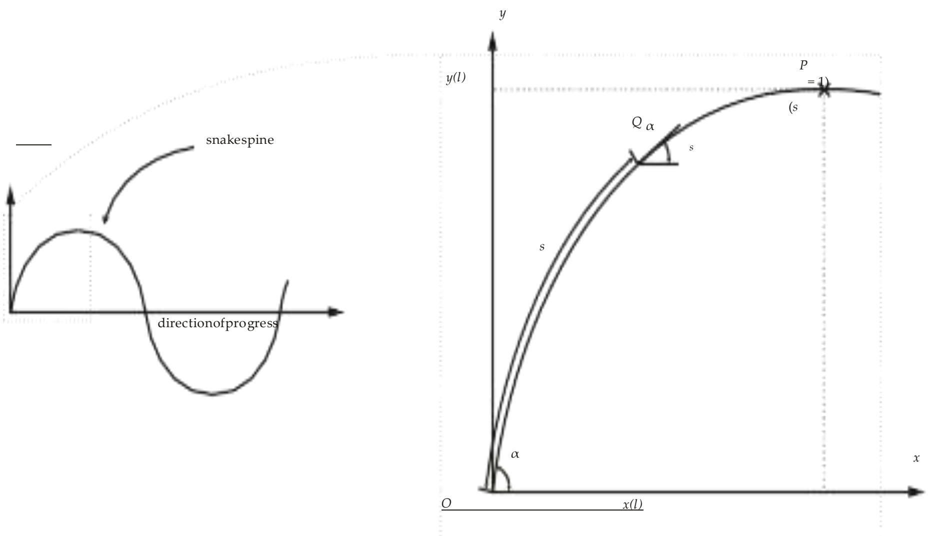

Fig. 6. The serpenoid curve [13].is a function of the distance along the curve s, the length of onequarterperiodofthe curve l, and the winding angel along the curve αs (s, l), where α = αs (0, l). Denote the tangential ci,t and normal ci,n frictional coefficient, between link i and the ground. The winding angle αs is determined by the link length, bending angles between adjacent links and the ratio ci,t/ci,n where the ratio also gives a lower bound for αs. The curvature of the serpenoid curve is given by

ρ (s) = α π sin ³ π s´. (11)

Except when otherwise specified, the common assumption is ci,t < ci,n. By adding a constant of turning motion c to (11), the relative reference angle for joint i is found from (11) to be (see e.g. [46])

φi = A sin (ωt + h (i) β)+ γ (12)

Except when otherwise specified, the common assumption is ci,t < ci,n. By adding a constant of turning motion c to (11), the relative reference angle for joint i is found from (11) to be (see e.g. [46])

φi = A sin (ωt + h (i) β)+ γ (12)

where A is the maximum amplitude of oscillation, β is the phase shift between adjacent links, γ determines the orientation of the snake robot, ω is the speed of the serpentine wave that propagates down the body of the snake robot and h (i) is a function that depends on the model of the snake robot,

e.g. h (i) = i 1 [19] or h (i) = i 2 [41]. The use of γ to change the heading of the snake is discussed and two alternative turning motions are presented in [46]. For snake robotswith 2 DOF revolute joints, a second reference signal for the vertical wave is needed [28]:

φi,v = Av sin (ωvt + h (i) βv + β0)+ γv . (13)

The phase difference between the horizontal and vertical wave is given by β0. The direction of locomotion when using lateral undulation (Av = 0) is controlled by γ, while β0 dominates the control of locomoting direction during sidewinding.

The wheeled ACM III has been used as an example in [14].

The angle between adjoining links and the wheel axles at the corresponding links is set 90◦ out of phase to locomote and avoid side slip of the wheels. The ACM III model has also beenused with fixed wheel axles where a Lyapunov based control method is proposed [35]. A wheeled snake robot able to move in 3D is presented in [47]. It is shown that for the robot to be controllable and observable, one needs 4 m n 2, where m is the number of wheels touching the ground. Some results on controllability are also given in [52].

Sinus-lifting has also been implemented [37], [45]. A 2D model incorporating a ground contact force which is a function of the curvature of the snake body is used in [37]. It is shown that the snake robot moves forward faster by sinus-lifting, than by lateral undulation.

Realsnakesuseobstaclessuchasstonesorindentationsinthe ground to aid locomotion during lateral undulation. Refer-ences [13], [53] utilize walls and large cylinders to generatepropulsive forces for a wheeled snake robot. A snake robotwithout wheels employs push-points, such as pegs, to createthetotalpropulsiveforceduringaformoflateralundulation[54]. Here, the ratio between lateral and longitudinal frictionisdisregarded,whichmakesitpossibletobuildandlocomotesnake robots without the friction property $c_{i,t}<c_{i,n}$, andhencewithoutwheels.Thejointsarenotbenttopushagainstthepegs;instead,thesnakerobothaslinearactuatorsoneachsideofthelinksthatpushoutfromthelink.

Sidewinding was performed with $c_{t,i}= c_{n,i}$in [38] andit was found that both γ and $\beta_0$controls the direction oflocomotion. A gait qualitatively similar to the sidewindinglocomotion has also been developed in [26] for 3D motiononaflatsurface.

B. Alternative Approachesto Locomotion

Whilemostoftheapproachestolocomotionpresentedaboverelyonthetangential/normalfrictionproperty,othershaveexploredalternativewaystolocomotewhereeitherthefriction model is isotropic or a purely kinematic model is used.Locomotionfora2-link,3-linkandmulti-link systemwith1DOFrevolutejointsareshownin[39],[40].Dryfrictionforcesbetweenthelinksandthegroundareassumed.Acombinationoffastandslowmovementsofthejointsisusedtomovethe2-,and3-linksystemtoanypointintheplane.Withverysmallvelocitiesandaccelerations,themulti-linksystemwasabletomovebypropagatingasinglewaveatthetimeconsistingof3to4linksforwardalongthesnake robots’body.

Inchworm-like motion has been shown with [22] and with-out[22],[29],[49]extensiblejoints.Whilethetwofirstreferences rely on slow speeds to avoid slipping, the latterpumpwaterbetweenthelinktoaddweighttotheparts that are not moving. This way, the ground contact force, and hencethefriction,onthemovingpartsarereduced.Figuresanddescriptionsofseveraltypesofgaitsfor2Dand3

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 4 Apr 2022, pp: 559-572 www.ijaem.net ISSN: 2395-5252

Dmotionofasnakerobotwithoutwheelsaregivenin[17].

V. DISCUSSION AND FUTURE RESEARCH TOPICS

Research on snake robots has increased the past ten years,buttherearestillmanychallengestofacebothonthemodelingandcontrolofsnakerobotsinordertomakethemableto locomote intelligently through unknown terrain. We haveseen in this article that various approaches to mathematicalmodeling of snake robots have been presented. Some focuspurely on the kinematic aspects of locomotion [14], [22], [52]while others also include the dynamics [3], [15], [19], [35],[41].Employingonlythekinematicmodelsimplifiesboththe model and the analysis of locomotion, and factors thatcontribute to locomotion have been highlighted [14], but thereis still no mathematical framework to prove locomotion of ageneral motion pattern. Passive wheels help defend the non-slip assumption of some kinematic models, but can it may bedifficulttocontrolthejointtorquessuchthatthewheelsdonot slip [3], [15]. A kinematic approach to locomotion withoutwheels is justified by assuming low velocities and sometimesalsocertainfrictionproperties(suchaslowfrictionwhile gliding forwards, but high friction when pushed backwards) [22].

Therearetwomainreasonstomodelasystem mathe-matically. One is that the model can be used to investigateanalytically how to control the system. The other reason is tosimulate the behavior of the system, for example for testingmotion patterns. Mathematical models including the dynamicsofmotionyieldamoreaccuratedescriptionof thebehaviorofthe system which is advantageous with respect to simulation,but the models may get very large and unwieldy. Thus, thesimplicityofananalyticalanalysissuffers.In2Dmodels,certain properties of the system help to simplify the model[21],butnotallofthesepropertiespersistin3Dmodels.Inthepaperspresented,mostmodelsare2D,butmovingina shattered building, for example is inherently a 3D experi-ence. 3D models have been presented [22], [26], [47] whichprovide some results on controllability and observability, butwithout including the dynamics. During the last two years, 3Ddynamic models based on the Newton-Euler equations havebeenpresented[28],[37].Therearetwomainreasonstomodelasystemmathe-matically. One is that the model can be used to investigateanalytically how to control the system. The other reason is tosimulate the behavior of the system, for example for testingmotion patterns. Mathematical models including the dynamicsofmotionyieldamoreaccuratedescriptionof thebehaviorofthe system which is advantageous with respect to simulation,but the models may get very large and unwieldy.may get very large and unwieldy.

Thus, thesimplicityofananalyticalanalysissuffers.In2Dmodels,certain properties of the system help to simplify the model[21],butnotallofthesepropertiespersistin3Dmodels.Inthepaperspresented,mostmodelsare2D,butmovingina shattered building, for example is inherently a 3D experience. 3D models have been presented [22], [26], [47] whichprovide some results on controllability and observability, butwithout including the dynamics. During the last two years, 3Ddynamic models based on the Newton-Euler equations havebeenpresented[28],[37].No3Dmodelshavebeen foundthat are developed using Lagrange’s equation, and this may bearesultofthattheinvarianceusedtosimplifytheequationsin 2D are no longer valid in 3D. Mathematical expressions forthejointtorquesandheadconfigurationarederivedin[37],but for the gait analysis, the model has been simplified toplanar movement with a varying ground contact force whichaffects the friction forces on each link. Hence, there are stillnumerouschallengesintheanalyticalinvestigation ofthedynamics of 3D locomotion. Regardless of the downsides ofdynamicmodeling,thedynamicsneedstobeconsideredinthecaseswhereslowlocomotionisunacceptableor whenwheels cannot be employed due to the nature of the surfacetravelled on. In such cases, friction and maybe even impactsneedto beconsideredandutilizedtoaidlocomotion.Theonlyimpactmodelfoundinliteratureregardingsnakerobotsis a linear spring-damper model [28], which results in a stiffmathematical model and thus a need to a low step length insimulations. Friction forces are modeled either by viscous orCoulomb friction, where the latter includes dry friction whichisessentialforsomegaits[40]. Thedirectionalfrictionpropertybetweenthebellyofthe snakeandtheground,mostoftenimplementedonsnake robots, cannot always be realized for travel on a variety ofsurfaces.Planarlocomotionutilizingdifferencesbetweenstaticanddynamicfriction[40]andexploitingpegsontheground [54]arebeinginvestigated,andthereisstillagreatamou nt of researchtobedoneinthisarea. Basedontheabovediscussion,thefollowingfutureresearchtopicsareproposed:1)Developamathematicalframeworkto help develop and prove general motion patterns, 2) Provecontrollability and observability while including the dynamicsofthesnakerobot,3)Developmotionpatterns thatareindependent of ground conditions, and 4) Find ways to betterusepegsorotherobstaclestoimprovelocomotion speed.

V.CONCLUSION

The use of snake-like robots has increased dramatically for rescue, surveillance and for spy purposes.A snake robot has limited payload capability, poor powerefficiency and a high number of degrees of freedom. Nevertheless,thesnakerobotexhibitsgreatterrainabilityandhasthecapabilityofinspectingnarrowplaces.Itcanalsobemadeveryrobusttodirtanddustbycoveringthe robot completely with a shell. The decision was made based on studies conducted over previous researches. Here an effort is done to implement a hybrid model of adaptive locomotion. This is employed by integrating various techniques and analysis methods into a single prototype

REFERENCES

[1] J.Gray,“Themechanismoflocomotioninsnakes,”J.Exp.Biol.,vol.23,no.2,pp.101–120,1946.

[2] S.Hirose, “Active code mechanism no.3"ACMIII",”2003.[Online].Available:http://www-robot.mes.titech.ac.jp/robot/snake/acm3/acm3_e.html

[3] P. Wiriyacharoensunthorn and S. Laowattana, “Analysis and design of amulti-link mobile robot (serpentine),” in Proc. IEEE Int. Conf. Robotics,Intelligent Systems and Signal Processing, vol. 2, December 2002, pp.694–699.

[4] M. Lewis and D. Zehnpfennig, “R7: A snake-like robot for 3-d visualinspection,” in Proc. Int. Conf. Intelligent Robots and Systems, vol. 2,MunichGermany,September1994,pp.1310–1317.

[5] R.WorstandR.Linnemann,“Constructionand operationofasnake-like robot,” in Proc. IEEE Int. Joint Symp. Intelligence and Systems,Rockville,MDUSA,November1996, pp.164–169.

[6] S.Ma,“Analysisofsnakemovementformsforrealizationofsnake-like robots,” in Proc. IEEE Int. Conf. Robotics and Automation, vol. 4,Detroit,MIUSA,May1999,pp.3007–3013.

[7] S.Ma,H.Araya,andL.Li,“Developmentofacreepingsnake-robot,”in Proc. IEEE Int. Symp. Computational Intelligence in Robotics andAutomation,2001,pp.77–82.

[8] Y. Lu, S. Ma, B. Li, and L. Chen, “Ground condition sensing of a snake-like robot,” in Proc. IEEE Int. Conf. Robotics, Intelligent Systems andSignalProcessing,vol.2,October2003,pp. 1075–1080.

[9] L.XinyuandF.Matsuno,“Controlofsnake-likerobotbasedonkinematic model with image sensor,” in Proc. IEEE Int. Conf. Robotics,Intelligent Systems and Signal Processing, vol. 1, October 2003, pp.347–352.

[10] T.Kamegawa,T.Yarnasaki,H.Igarashi,andF. Matsuno,“Developmentofthesnake-likerescuerobot"kohga",”inProc.IEEEInt.Conf.RoboticsandAutomation,vol.5,April2004, pp.5081–5086.

[11] G. Chirikjian and J. Burdick, “A modal approach to hyper-redundantmanipulator kinematics,” IEEE Trans. Robot. Autom., vol. 10, no. 3, pp.343–354,June1994.

[12] P. Krishnaprasad and D. Tsakiris, “G-snakes: Nonholonomic kinematicchains on lie groups,” in Proc. 33rd IEEE Conf. Decision and Control,vol.3,LakeBuenaVista,FLUSA,December1994,pp.2955–2960.

[13] S. Hirose, Biologically Inspired Robots: Snake-Like Locomotors andManipulators.Oxford:OxfordUniversityPress,1993.

[14] J.Ostrowskiand J. Burdick,“Gait kinematics fora serpentinerobot,”inProc.IEEEInt.Conf.RoboticsandAutomation,vol.2,April1996,pp.1294–1299.

[15] S.Ma,N.Tadokoro,B.Li,andK.Inoue,“Analysisofcreepinglocomotionofasnakerobotonaslope,”inProc.IEEEInt.Conf.RoboticsandAutomation,September2003,pp.2073–2078.

[16] Y. Shan and Y. Koren, “Design and motion planning of a mechanicalsnake,” IEEE Trans. Syst. Man Cyb., vol. 23, no. 4, pp. 1091–1100,July-August1993.

[17] H.OhnoandS.Hirose,“Designofslimslimerobotanditsgaitof locomotion,” in Proc. IEEE/RSJ Int. Conf. Intelligent Robots andSystems,vol.2,November2001,pp.707–715.

[18] S. Ma, “Analysis of creeping locomotion of a snake-like robot,”

Adv.Robotics,vol.15,no.2,pp.205–224,2001.
[19] M.Saito,M.Fukaya,andT.Iwasaki,“Serpentinelocomotionwithrobotic snakes,” IEEE Contr. Syst. Mag., vol. 22, no. 1, pp. 64–81,February2002.
[20] I. Grabec, “Control of a creeping snake-like robot,” in Proc. 7th Int.WorkshoponAdvancedMotionControl,July2002,pp.526–513.
[21] K. McIsaac and J. Ostrowski, “Motion planning for anguilliform loco-motion,”IEEETrans.Robot.Autom.,vol.19,no.4,pp.637–625,August2003.
[22] G. Chirikjian and J. Burdick, “The kinematics of hyper-redundant robotlocomotion,” IEEE Trans. Robot. Autom., vol. 11, no. 6, pp. 781–793,December1995.
[23] C.Mattison,TheEncyclopaediaofSnakes.London:CassellPaperbacks,2002.
[24] R. Bauchot, Snakes: A Natural History.Sterling Publishing Company,1994.
[25] K. J. Dowling, “Limbless locomotion. learing to crawl with a snakerobot,”Ph.D. dissertation,CarnegieMellonUniversity,December 1997.
[26] J. Burdick, J. Radford, and G. Chirikjian, “A ’sidewinding’ locomotiongait for hyper-redundant robots,” in Proc. IEEE Int. Conf. Robotics andAutomation,May1993,pp.101–106.
[27] R. M. Murray, Z. Li, and S. S. Sastry, A Mathematical Introduction toRoboticManipulation,1sted.Florida,USA:CRCPress,1994.
[28] P.Liljebäck,Ø.Stavdahl,andK.Y.Pettersen,“ Modularpneumaticsnakerobot: 3D modelling, implementation and control,” in Proc. 16th IFACWorldCongress,July2005.
[29] G.Poi,C.Scarabeo,andB.Allotta,“Travelingwavelocomotionhyper-redundant mobile robot,” in Proc. IEEE Int. Conf. Robotics andAutomation,vol.1,May1998,pp.418–423.
[30] G. S. Chirikjian, “Theory and applications of hyper-redundant roboticmanipulators,” Ph.D. dissertation, California Institute of Technology,Pasadena,California,1992.
[31] G.Chirikjian and J.Burdick,“Kinematics ofhyper-redundantrobotlocomotion with applications to grasping,” in Proc.IEEEInt. Conf.RoboticsandAutomation,April1991,pp. 720–725.
[32] I. Kolmanovsky and N. McClamroch, “Developments in nonholonomiccontrol problems,” IEEE Contr. Syst. Mag., vol. 15, no. 6, pp. 20–36,December1995.
[33] A. M. Bloch, J. Baillieul, P. Crouch, and J. Marsden, NonholonomicMechanicsandControl.Springer-Verlag,2003.
[34] S.KellyandR.M. Murray,“Geometricphases androboticlocomotion,” J.RoboticSystems,vol.12,no.6,pp.417–431,1995.
[35] P. Prautsch and T. Mita, “Control and analysis of the gait of snakerobots,”inProc.IEEEInt.Conf.ControlApplications,KohalaCoast,HIUSA,1999,pp.502–507.
[36] O. Egeland and J. T. Gravdahl, Modeling and Simulation for AutomaticControl.Trondheim,Norway:MarineCybernetics,2002.
[37] S.Ma,Y.Ohmameuda,andK.Inoue,“Dynamic analysisof3-dimensional snake robots,” in Proc. IEEE/RSJ Int. Conf. On IntelligentRobotsandSystems,2004,pp.767–772.
[38] P.Liljebäck,“Modularsnake-robot:Modeling,implementationandcontrol of a modular and pressure based snake-robot,” Master’s thesis,Norwegian University of Technology and Science, Trondheim, Norway,2004,inNorwegian.
[39] F. Chernousko, “Snake-like motions of multibody systems over a roughplane,” in Proc. 2nd Int. Conf. Control of Oscillations and Chaos, July2000,pp.321–326.
[40] Chernousko, “Modelling of snake-like locomotion,” Appl. Math. Com-put.,vol.164,no.2,pp.415–434,May2005.
[41] K. McIsaac and J. Ostrowski, “A geometric approach to anguilliformlocomotion: Modelling of an underwater eel robot,” in Proc. IEEE Int.Conf.RoboticsandAutomation,vol.4,May 1999,pp.2843–2848.
[42] ——, “Motion planning for dynamic eel-like robots,” in Proc IEEE Int.Conf.RoboticsandAutomation,vol.2,2000,pp.1695–1700.
[43] J.Ayers,C.Wilbur,andC.Olcott,“Lampreyrobots,”inProc.Int.Symp.AquaBiomechanisms,2000.
[44] A. Bloch, P. Krishnaprasad, J. Marsden, and R. Murray, “Nonholonomicmechanical systems with symmetry,” California Institute of Technology,Tech.Rep.,1995.
[45] M.MoriandS.Hirose,“Three-dimensionalserpentinemotionandlateralrolling by active cord mechanism ACM-R3,” in Proc. IEEE/RSJ Int.Conf.IntelligentRobotsandSystems,2002,

International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 4 Apr 2022, pp: 559-572 www.ijaem.net ISSN: 2395-5252

pp.829–834.
[46] C. Ye, S. Ma, B. Li, and Y. Wang, “Turning and side motion of snake-like robot,” in Proc. IEEE Int. Conf. Robotics and Automation, vol. 5,2004,pp.5075–5080.
[47] S. Ma, Y. Ohmameuda, K. Inoue, and B. Li, “Control of a 3-dimensionalsnake-like robot,” in Proc. IEEE Int. Conf. Robotics and Automation,vol.2,Taipei,Taiwan,September 2003,pp.2067–2072.[Online].Available:http://dx.doi.org/10.1109/ROBOT.2003.1241898
[48] Z. Bayraktarouglu, F. Butel, P. Blazevic, and V. Pasqui, “A geometricalapproach to the trajectory planning of a snake-like mechanism,” in Proc.IEEE/RJS Int. Conf. Intelligent Robots and Systems, October 1999, pp.1322–1327.
[49] D. Rincon and J. Sotelo, “Ver-Vite: Dynamic and experimental analysisforinchwormlikebiomimeticrobots,”IEEERobot.Autom.Mag.,vol.10,no.4,pp.53–57,December2003.
[50] M.Nilsson,“Snakerobotfreeclimbing,”inProc.IEEEInt.Conf.RoboticsandAutomation,vol.4,April1997,pp.3415–3420.
[51] ——,“Snakerobot-freeclimbing,”IEEEContr.Syst.Mag.,vol.18,no.1,pp.21–26,February1998.
[52] J. Ostrowski and J. Burdick, “The geometric mechanics of undulatoryrobotic locomotion,” Int. J. Robot. Res., vol. 17, no. 7, pp. 683–701,1998.
[53] S. Hirose and Y. Umetani, “Kinematic control of active cord-mechanismwith tactile sensors,” in Proc. 2nd RoMAnSy Symp., Warsaw, 1976, pp.241–252.
[54] Z. Bayraktaroglu and P. Blazevic, “Understanding snakelike locomotionthrough a novel push-point approach,” J. Dyn. Syst. - Trans. ASME, vol.127,no.1,pp.146–152,March2005.